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Higher Dimensional Λ-CDM Universe: A Phenomenological Approach with Many Possibilities

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Abstract In this paper we investigate the Λ-CDM universe by selecting the specific time dependent form of Λ, viz. $\dot{\Lambda} \propto H^3$, in the context of 5-dimensional space time. Time-dependent form of equation of state parameter w is derived along with a possible signature flip of deceleration parameter q. It is also observed that a present age of the Universe, calculated for some specific values of parameter agrees very well with the observational data.

Keywords Dark energy · Variable Λ · Λ-CDM cosmology

1 Introduction

Λ-CDM or Lambda-CDM is an abbreviation for Lambda-Cold Dark Matter. Cold dark matter (or CDM) is a refinement of the big bang theory that contains the additional assumption that most of the matter in the Universe consists of material that cannot be observed by its electromagnetic radiation and hence is dark while at the same time the particles making up this matter are slow and hence are cold. It is frequently referred to as the concordance model of big bang cosmology, since it attempts to explain cosmic microwave background observations, as well as large scale structure observations and supernovae observations of the accelerating expansion of the universe. It is the simplest known model that is in general

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agreement with observed phenomena. Moreover, Λ-CDM says nothing about the fundamental physical origin of dark matter, dark energy and the nearly scale-invariant spectrum of primordial curvature perturbations: in that sense, it is merely a useful parameterisation of ignorance. An advantage of Λ-CDM model is that it assumes a nearly scale-invariant primordial perturbations and a Universe with no spatial curvature. These were also predicted by inflationary scenario [1–5].

In astrophysics and cosmology, dark matter is a hypothetical form of matter of unknown composition that does not emit or reflect enough electromagnetic radiation to be observed directly, but whose presence can be inferred from gravitational effects on visible matter. According to present observations of structures larger than galaxies, as well as Big Bang cosmology, dark matter accounts for the vast majority of mass in the observable universe. The observed phenomena consistent with dark matter observations include the rotational speeds of galaxies, orbital velocities of galaxies in clusters, gravitational lensing of background objects by galaxy clusters such as the Bullet cluster, and the temperature distribution of hot gas in galaxies and clusters of galaxies. Dark matter also plays a central role in structure formation and galaxy evolution, and has measurable effects on the anisotropy of the cosmic microwave background. All these lines of evidence suggest that galaxies, clusters of galaxies, and the universe as a whole contain far more matter than that which interacts with electromagnetic radiation: the remainder is called the "dark matter component".

The dark matter component has vastly more mass than the "visible" component of the universe. At present, the density of ordinary baryons and radiation in the universe is estimated to be equivalent to about one hydrogen atom per cubic metre of space. Only about 4% of the total energy density in the universe (as inferred from gravitational effects) can be seen directly. About 22% is thought to be composed of dark matter. The remaining 74% is thought to consist of dark energy, an even stranger component, distributed diffusely in space. Some hard-to-detect baryonic matter makes a contribution to dark matter, but constitutes only a small portion. Determining the nature of this missing mass is one of the most important problems in modern cosmology and particle physics. It has been noted that the names "dark matter" and "dark energy" serve mainly as expressions of human ignorance, much as the marking of early maps with "terra incognito".

Λ (Lambda) stands for the cosmological constant which is a dark energy term that allows for the current accelerating expansion of the universe. The cosmological constant is often described in terms of Ω_Λ, the fraction of the energy density of a flat Universe in the form of the cosmological constant. Currently, 0.74, implying 74% of the energy density of the present universe is in this form.

Mukhopadhyay et al. [6] selected a time dependent model of Λ and derived the time dependent form of equation of state parameter w in general theory of relativity. The present work is done with this back ground in mind in the context of Kaluza-Klein theory of gravitation.

In this paper by selecting $\dot{\Lambda} \propto H^3$, we investigate a time involving equation of state parameter ω along with a possible signature flip of deceleration parameter q in the context of Kaluza-Klein theory of gravitation. This work is an extension of the work obtained earlier by Mukhopadhyay et al. [6] in the context of Kaluza-Klein theory of relativity.

2 Model and Field Equations

Let us consider the 5D Robertson-Walker metric

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{(1-kr^2)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] + A^2(t)d\psi^2, \tag{1}$$

where $R(t)$ is the scale factor and $k = 0, -1$, or $+1$ is the curvature parameter for flat, open and closed universe, respectively. The universe is assumed to be filled with distribution of matter represented by energy-momentum tensor of a perfect fluid

$$T_{ij} = (p + \rho)u_i u_j - p g_{ij}, \tag{2}$$

where ρ is the energy density of the cosmic matter and p is its pressure and u_i is the five-velocity vector such that $u_i u^j = 1$.

The Einstein field equations are given by

$$R_{ij} - \frac{1}{2} g_{ij} R = -8\pi G\left[T_{ij} - \frac{\Lambda(t)}{8\pi G} g_{ij}\right], \tag{3}$$

where the cosmological term Λ is time-dependent and c, the velocity of light in vacuum is assumed to be unity.

For the metric (1) with energy momentum tensor (2) along with $A(t) = R^n$, the field equations (3) yields three independent equations

$$8\pi G\rho = 3\left[(n+1)\frac{\dot{R}^2}{R^2} + \frac{k}{R^2}\right] - \Lambda(t), \tag{4}$$

$$8\pi G p = -(n+2)\frac{\ddot{R}}{R} - (n^2 + n + 1)\frac{\dot{R}^2}{R^2} - \frac{k}{R^2} + \Lambda(t), \tag{5}$$

$$8\pi G p = -3\left(\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2}\right) + \Lambda(t), \tag{6}$$

where dot denotes derivative with respective to t.

The above field equations can also be expressed as

$$3(n+1)H^2 + \frac{3k}{R^2} = 8\pi G\rho + \Lambda(t), \tag{7}$$

$$3(n+1)H^2 + 3(n+1)\dot{H} = -8\pi G(t)[(n+1)p + \rho] - 3n\frac{k}{R^2} + n\Lambda(t), \tag{8}$$

where G, ρ and p are the gravitational constant, matter energy density and pressure respectively and the Hubble parameter H is related to the scale factor by $H = \frac{\dot{R}}{R}$. In the present work, G is assumed to be constant.

The conservation equation for variable Λ and constant G is given by

$$\dot{\rho} + (3+n)(\rho + p)H = -\frac{\dot{\Lambda}}{8\pi G}. \tag{9}$$

Let us consider a barotropic equation of state

$$p = w\rho, \tag{10}$$

where w is the barotropic index which has been considered here as time-dependent.

Using (10) we get from (9)

$$8\pi G\dot{\rho} + \dot{\Lambda} = -(3+n)8\pi G(1+w)\rho H. \tag{11}$$

After differentiating (7) with respect to t we get for a flat Universe ($k = 0$)

$$-4\pi G\rho = \frac{3(n+1)\dot{H}}{(3+n)(1+w)}. \tag{12}$$

Let us, also consider $\dot{\Lambda} \propto H^3$, so that

$$\dot{\Lambda} = AH^3, \tag{13}$$

where A is a proportional constant.

Using (10), (12) and (13), we get from (8)

$$\frac{D_1}{H^3}\frac{d^2H}{dt^2} + \frac{2}{H^2}\frac{dH}{dt} = \frac{n}{3(n+1)}A, \tag{14}$$

where, $D_1 = [\frac{(3+n)(1+w)-2(1+(n+1)w)}{(3+n)(1+w)}]$.

If we put $\frac{dH}{dt} = P$, then (14) reduces to

$$\frac{dP}{dH} + \frac{2H}{D_1} = \frac{n}{3(n+1)}\frac{AH^3}{D_1 P}. \tag{15}$$

In the following section we now solve (15) under some specific assumptions.

3 Solutions

3.1 For $A = 0$

In this case (15) reduces to

$$\frac{dP}{dH} + \frac{2H}{D_1} = 0. \tag{16}$$

Solving (16) for $R(t)$, $\rho(t)$ and $H(t)$ we get

$$R(t) = C_1 t^{D_1}, \tag{17}$$

$$H(t) = \frac{D_1}{t}, \tag{18}$$

$$\rho(t) = \frac{3(n+1)D_1}{4\pi G(3+n)(1+w)}\frac{1}{t^2}, \tag{19}$$

where, C_1 is a constant.

From (18) the deceleration parameter q can be expressed as

$$q = -\left(1 + \frac{\dot{H}}{H^2}\right). \tag{20}$$

From (20) it is observed that for an accelerating Universe, $w < -\frac{1}{(n+1)}$ and it is clear that from (17)–(19) w cannot be equal to -1. The sought for signature flipping of q can be obtained from (20) if one consider w as time-dependent.

If H_0 and t_0 be the present values of H and t, then from (18) we can write,

$$t_0 = \frac{D_1}{H_0}. \tag{21}$$

3.2 For $\frac{1}{D_1} = -\frac{3P}{H}$

In this case (15) becomes,

$$\frac{dP}{dH} - 6P = -a_0 A H^2. \tag{22}$$

Solving (22) we get

$$R(t) = C_2 e^{-\frac{t}{6}} (\sec Bt)^{\frac{1}{6B}}, \tag{23}$$

$$H(t) = \frac{1}{6}(\tan Bt - 1), \tag{24}$$

$$\Lambda(t) = \frac{B}{6a_0}\left[\frac{1}{2B}\tan^2 Bt + \frac{2}{B}\log(\sec Bt) - \frac{3}{B}\tan Bt + 2t\right], \tag{25}$$

$$\rho(t) = \frac{(n+1)(X(t) - 2n)B \sec Bt \tan Bt}{16\pi G n(3+n)}, \tag{26}$$

$$w(t) = -\left[1 + \frac{2n}{X(t) - 2n}\right], \tag{27}$$

where C_2 is a constant of integration, $a_0 = \frac{2n}{(n+1)}$, $B = \frac{a_0 A}{36}$ and

$$X(t) = \left[(3+n)(1 + \frac{\tan Bt - 1}{3B\sec^2 Bt}) - 2\right].$$

For physically valid H, $\tan Bt > 1$. Then (26) implies that for a positive B, w must be less than -1 as in the case of phantom energy. On the other hand if $B < 0$, then w can be greater than -1 as well.

Again, using (24) we get

$$q = -\left[1 + \frac{6B\sec^2 Bt}{(\tan Bt - 1)^2}\right]. \tag{28}$$

We find from (28) if $B < 0$, a signature flipping of q is possible. This value is similar to the value obtained by Mukhopadhyay et al. [6] in the four dimensional general theory of relativity. So, the merit of this case lies the fact that the same change of sign of q can be obtained here by using a time-dependent form of w and not making any special assumption on q directly as was done by Banerjee and Das [7]. This shows that w is a key ingredient of cosmic evolution.

3.3 For $\frac{1}{D_1} = -\frac{P}{2H^2}$

With the above assumption, (15) becomes,

$$\frac{dP}{dH} - \frac{P}{H} = -\frac{a_0 A}{6} H. \tag{29}$$

Solving (29) we get our solution set as

$$R(t) = C_3 t^{\frac{6}{a_0 A}}, \tag{30}$$

$$H(t)=\frac{6}{a_0 A t}, \tag{31}$$

$$\rho(t)=\frac{9(n+1)(2n-A_0)}{4a_0 A\pi G n(3+n)}\frac{1}{t^2}, \tag{32}$$

$$\Lambda(t)=-\frac{108}{a_0^3 A^2}\frac{1}{t^2}, \tag{33}$$

$$w(t)=-\left[1+\frac{2n}{A_0-2n}\right], \tag{34}$$

where C_3 is an integration constant and $A_0=[(3+n)(1-\frac{12}{a_0 A})-2]$.

Thus, from the above solution it is observed that scale factor admits a power law solution, H varies inversely as t and ρ as well as Λ follow the well known inverse square law with t. This type of solution was obtained by Ray et al. [8] for $\Lambda\sim(\frac{\dot{R}}{R})^2$, $\Lambda\sim\frac{\ddot{R}}{R}$ and $\Lambda\sim\rho$ in the context of higher dimensional space time.

Again, in this case

$$q=-\left(1-\frac{a_0 A}{6}\right). \tag{35}$$

From (35) it is observed that for a negative a_0 and for a positive A, the Universe expands with a constant acceleration. For $A=-\frac{6}{a_0}$ the amount of acceleration is -2. So in case of the phenomenological model $\dot{\Lambda}\sim H^3$, the deceleration parameter q does not show any change in sign during cosmic evolution in the context of Kaluza-Klein theory of gravitation.

4 Conclusion

This work has thus generalised to higher dimension the well known results in four dimensional space time. It is found that there may be a significant difference, in principal at least to the analogous situation in four dimensional space time. The main objectives of the present work were to search for a signature flip of q and find a time -dependent expression for the equation of state parameter. By selecting a time-dependent form of cosmological parameter Λ through some analytical study, it has been possible to show that a change in sign of deceleration parameter can be achieved under the special assumption $\frac{1}{D_1}=-\frac{3P}{H}$ in the context of higher dimensional space time. It has been also possible to derive time-dependent expression for equation of state parameter w.

Equation (15) shows that for $w=-1$, H grows linearly with time which does not feet with present cosmological scenario. It is also found that w can be less than -1 as well as which is compatible with SN Ia data [9] and SN Ia data with CMB anisotropy and galaxy-cluster statistics [10].

We obtained the solution in Sect. 3.1 for $A=0$ and in Sect. 3.3, for $\frac{1}{D_1}=-\frac{P}{2H^2}$ and it is observed that the scale factor admits the power law solution H, varies inversely as t and ρ varies as inverse square law with t. In Sect. 3.2, for the case $\frac{1}{D_1}=-\frac{3P}{H}$ from (28) it is observed that a signature of flipping of q is possible if $B<0$.

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References

1. Mukhanov, V.F., Chibisov, G.B.: JETP **33**, 532 (1981)
2. Guth, A.H., Pi, S.Y.: Phys. Rev. Lett. **49**, 1110 (1982)
3. Hawking, S.W.: Phys. Rev. Lett. B **115**, 295 (1982)
4. Starobinsky, A.A.: Phys. Rev. Lett. B **117**, 175 (1982)
5. Bardeen, J., Steinhardt, P.J., Turner, M.S.: Phys. Rev. D **28**, 679 (1983)
6. Mukhopadhyay, U., Ray, S., Dutta Choudhury, S.B.: Int. J. Mod. Phys. D **17**, 301 (2008)
7. Banerjee, N., Das, S.: Gen. Relativ. Gravit. **37**, 1695 (2005)
8. Ray, S., Mukhopadhyay, U., Meng, X.H.: Gravit. Cosmol. **13**, 142 (2007)
9. Knop, R.A., et al.: Astrophys. J. **598**, 102 (2003)
10. Tegmark, M., et al.: Astrophys. J. **606**, 702 (2004)